EX.1)

a) RAPPEL: Monotonie et signe de la fonction exponentielle (e^x)

x	$-\infty$		0		$+\infty$
e^x	0	↗	1	↗	$+\infty$

Pour tot reel x, $e^x>0$. Ainsi, $e^x+1>0$, par croissance de la fonction affine $x \to x+1$ sur $\mathbb{R}$. Comme $e^x+1 \neq 0$, f est définie sur $\mathbb{R}$.

b)

$$f(x)+f(-x)=\frac{2}{e^x+1}+\frac{2}{e^{-x}+1}=2\left(\frac{1}{e^x+1}+\frac{1}{e^{-x}+1}\right)=2\left(\frac{e^{-x}+1+e^x+1}{(e^x+1)(e^{-x}+1)}\right)$$

$$=2\left(\frac{e^{-x}+e^x+2}{e^xe^{-x}+e^x+e^{-x}+1}\right)=2\left(\frac{e^{-x}+e^x+2}{1+e^x+e^{-x}+1}\right)=2\left(\frac{e^{-x}+e^x+2}{e^x+e^{-x}+2}\right)=2$$

EX.2)

a) ln2-ln6+ln12 = ln[(2.12)/6)] = ln4

ou

ln2-ln6+ln12 = ln2-(ln2+ln3)+ln3+ln4 = ln2-ln2-ln3+ln3+ln4 = ln4

b) ln1+ln2+ln3+.....+ln(n-1)+ln(n) = ln[1.2.3.......(n-1).n)] = ln (n!)

c) ln25-ln10-ln15 = ln[25/(10.15)] = ln (25/150) = ln(1/6) = ln(6^{-1}) = -ln6

ou

ln25-ln10-ln15 = ln 5^2 - (ln5+ln2) – (ln5+ln3) = 2ln5-ln5-ln2-ln5-ln3 = -(ln2+ln3)

=-ln(2.3) = -ln6

d) $\log_2 20-\log_2 10+\log_3 60-\log_3 20 = \log_2(20/10)+\log_3(60/20) = \log_2 2+\log_3 3 = 1+1 = 2$

e) $\log_6 25-\log_6 10-\log_6 15 = \log_6[25/(10.15)] = \log_6(25/150) = \log_6(1/6) = \log_6(6^{-1})$

$= -\log_6 6 = -1$

EX.3) logx+log2=1 => log(x.2) = 1 => log(2x) = 1 => 2x=10 => x=5

EX.4) $\log_5 3 + \log_5 a = \log_5(3.a) = \log_5(3a) = 1$ => 3a=5 => a=5/3

EX.5) $\log_2 a = \log_{1/2} b = -\log_2 b$ => $\log_2 a = \log_2 b^{-1}$ => $a = b^{-1} = 1/b$

$\log_{10}(ab) = \log_{10}[a.(1/a)] = \log_{10}1 = 0$

EX.6)

a)

$$log_2[log_3(5x+6)] = 2 \;=>\; log_3(5x+6) = 2^2 \;=>\; log_3(5x+6) = 4 \quad => (5x+6) = 3^4$$

$$=> \; 5x+6 = 81 \;=>\; 5x = 75 \;=>\; x = 15$$

$$log20 - \log(x-1) = 1 \;=>\; \log\left(\frac{20}{x-1}\right) = 1 \;=>\; \frac{20}{x-1} = 10^1 \;=>\; 20 = 10x - 10$$

$$=> \; 10x = 30 \;=> x = 3$$

c)

$$log_{10}(x+1) - log_{10}x = 3 \;=>\; log_{10}\left(\frac{x+1}{x}\right) = 3 \;=>\; \frac{x+1}{x} = 10^3 \;=>\; x+1 = 1000x$$

$$=> \; 999x = 1 \;=>\; x = \frac{1}{999}$$

d)

$$log_2[2log_3(3log_4(x+2))] = 1 \;=>\; 2log_3(3log_4(x+2)) = 2 \;=>\; log_3(3log_4(x+2)) = 1$$

$$=> 3log_4(x+2) = 3 \;=>\; log_4(x+2) = 1 \;=>\; x+2 = 4 \;=>\; x = 2$$

e)

$$x.log_2 3 - (\sqrt{x}+1).log_4 3 = 0 \;=>\; x.log_2 3 = (\sqrt{x}+1).log_4 3 \;=>\; \frac{log_2 3}{log_4 3} = \frac{(\sqrt{x}+1)}{x}$$

$$=> \quad \frac{log_2 3}{\frac{1}{2}log_2 3} = \frac{(\sqrt{x}+1)}{x} \;=>\; 2 = \frac{(\sqrt{x}+1)}{x} \;=>\; x = 1$$

f) $9^{x+1} + 3^{x+1} - 6 = 0$ = $9.9^x+3.3^x-6=0$ Soit $t=3^x$.

On obtient: $9t^2+3t-6 = 0$ => $(9t-6)(t+1)=0$ => $t=2$ ou $t=-1$

$3^x \neq -1$ (pas de solution)

$3^x=6/9=2/3$ => $x = \log_3(2/3)$

g)

$$log_2 3x + log_4 x^2 = 2 \;\; => \;\; log_2 3x + log_{2^2} x^2 = 2 \;\; => \;\; log_2 3x + \frac{1}{2} 2log_2 x => \;\; log_2(3x.x) = 2$$

$$log_2(3x^2) = 2 \;\; => \;\; 3x^2 = 4 \;\; => \;\; x^2 = \frac{4}{3} \;\; => \;\; x = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

h) $log_8[log_9(\sqrt{x+1})] = -\frac{2}{3} \;\; => \;\; log_9(\sqrt{x+1}) = 8^{\frac{-2}{3}} = (2^3)^{(-2/3)}$

$$log_9(\sqrt{x+1}) = 2^{-2} = \frac{1}{4} \;\; => \;\; \sqrt{x+1} = 9^{\frac{1}{4}} \;\; => \;\; \sqrt{x+1} = (3^2)^{\frac{1}{4}} \;\; => \;\; (x+1)^{\frac{1}{2}} = 3^{\frac{1}{2}}$$

$$=> \;\; x + 1 = 3 \;\; => \;\; x = 2$$

i)

$$log_3(log_2 32) = log_9 x \;\; => \;\; log_3(log_2 2^5) = log_{3^2} x \;\; => \;\; log_3(log_2 2^5) = \frac{1}{2} log_3 x$$

$$=> \;\; log_3 5 = log_3 x^{\frac{1}{2}} \;\; => \;\; x^{\frac{1}{2}} = 5 \;\; => \;\; x = 25$$

j) $logx + 2\log(1/x) = log8 - 2logx \;\; => \;\; logx + 2\log(x^{-1}) + 2logx = log8$

$=> \;\; logx - 2logx + 2logx = log8 \;\; => \;\; logx = log8 \;\; => \;\; x = 8$

EX.7

$$log_7(2x-7) - log_7(x-2) = 0 \;\; => \;\; log_7 \frac{2x-7}{x-2} = 0 \;\; => \;\; \frac{2x-7}{x-2} = 1 \;\; => \;\; x = 5$$

Ou, $log_7(2x-7) - log_7(x-2) = 0 \;\; => \;\; log_7(2x-7) = log_7(x-2) \;\; => 2x - 7 = x - 2$

$$=> \;\; x = 5$$

Alors, $log_5 x = log_5 5 = 1$